

## Elastic constants of transversely isotropic media from constrained aperture traveltimes

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### ABSTRACT

The elastic constants that control  $P$ - and  $SV$ -wave propagation in a transversely isotropic media can be estimated by using  $P$ - and  $SV$ -wave traveltimes from either crosswell or VSP geometries. The procedure consists of two steps. First, elliptical velocity models are used to fit the traveltimes near one axis. The result is four elliptical parameters that represent direct and normal moveout velocities near the chosen axis for  $P$ - and  $SV$ -waves. Second, the elliptical parameters are used to solve a system of four equations and four unknown elastic constants. The system of equations is solved analytically, yielding simple expressions for the elastic constants as a function of direct- and normal-moveout velocities. For  $SH$ -waves, the estimation of the corresponding elastic constants is easier because the phase velocity is already elliptical. The procedure, introduced for homogeneous media, is generalized to heterogeneous media by using tomographic techniques.

### INTRODUCTION

The effect of velocity anisotropy on wave propagation in homogeneous and heterogeneous media has been the subject of numerous publications. Careful forward modeling has helped interpreters understand how velocity anisotropy manifests itself in field data. Few attempts have been made, however, to estimate the parameters that describe the complexity of velocity anisotropy, namely, the elastic constants. Estimation of elastic constants is important for lithologic discrimination and fracture orientation, for revealing anisotropic properties of the medium that are not obvious in the data, and for providing further imaging or full-waveform inversion algorithms with background models that can be refined iteratively.

Elastic constants have been estimated primarily in the lab where the samples can be assumed to be homogeneous. When the assumption of homogeneity is not valid, the problem is more difficult because the effects of anisotropy and heterogeneity are coupled in the data. This coupling problem has been addressed by making prior assumptions about the nature of both the anisotropy and heterogeneity. For example, by assuming that the rock samples are homogeneous, Arts et al. (1991) estimate, from lab measurements, the 21 elastic constants that characterize a general triclinic system. By assuming isotropic velocities, McMechan (1983) estimates arbitrary spatial variations using 2-D and 3-D tomographic models. These two papers are examples of different trade-offs between velocity anisotropy and heterogeneity. On the one hand, the first paper uses the simplest model for heterogeneities (homogeneous) without making any assumption about the type of velocity anisotropy. On the other hand, the second paper uses the simplest model for the velocity (isotropic) without making any assumption about the nature of the heterogeneity.

A model commonly used to describe velocity anisotropy is transverse isotropy (TI). This approach is popular because transverse isotropy is the most common form of anisotropy in the subsurface. Several authors have addressed the issue of estimating elastic constants that characterize this type of anisotropy. White et al. (1983) estimate the five TI elastic constants of a homogeneous formation using a VSP geometry. Hake et al. (1984) approximate the traveltimes curves of layered models with a three-term Taylor series expansion in which the coefficients are functions of the elastic constants. Winterstein and Paulsson (1990) estimate the elastic constants from VSP and crosswell measurements in a medium with velocity linearly increasing with depth. Byun and Corrigan, (1990) propose an iterative model-based optimization scheme to invert traveltimes for the five elastic constants of a TI-layered media. More recently, Sena (1991) proposed a variant of this method in which all the calculations are performed analytically, without having to go

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through the semblance analysis needed in Byun and Corrigan's method.

Near the axes of symmetry, a TI medium looks like an elliptically anisotropic medium. This property of transverse isotropy is used by Muir (1990a) and Dellinger et al. (1993) to approximate *P*- and *SV*-wave slowness surfaces and impulse responses with ellipses fitted near *both* horizontal and vertical axes. In a later paper, Muir (1990b) suggests the transformation of the elliptical parameters into elastic constants by using well-known expressions that relate them (Levin, 1979; Levin, 1980). However, Muir implicitly assumes in this paper that traveltimes near both axes are available, which does not happen often.

Unfortunately, all the preceding methods fail when the data are not wide aperture or when measurements along both axes are not available. These conditions commonly exist in VSP and crosswell experiments.

In this paper, I show how to obtain the elastic constants that control *P*- and *SV*-wave propagation in TI media from limited aperture traveltimes, either from VSP or from crosswell geometries. I start by fitting the traveltimes for *P*- and *SV*-waves with elliptical time-distance relations near a single axis (either vertical or horizontal). The result is four velocities: two based on the time-of-arrival and distance along a symmetry axis (the direct velocities) and two based on the differential traveltime and differential distance as the direction is perturbed (the normal moveout-NMO-velocities). These four velocities are used to solve analytically a system of four equations and four unknown elastic constants. Since the procedure is based on fitting the data with elliptical velocity models, it is exact when estimating elastic constants from *SH*-wave traveltimes.

The data aperture is constrained in two different ways. First, it should not be too small to ensure that there is enough curvature to estimate the normal moveout velocities. Second, it should not be too wide to ensure that the elliptical fit remains accurate for the given wave type. I show that there is an intermediate range of ray angles that satisfy these

The equations I use in this paper to transform elastic constants into elliptical parameters (forward mapping) are not new. They are the same as the ones summarized by Muir (1990b), which can also be found in Levin (1979) and Levin (1980) for near-horizontal and near-vertical propagation, and Thomsen (1986) for near-vertical propagation. What is new is the simultaneous solution of these equations near each axis to obtain elastic constants as a function of elliptical parameters (inverse mapping). The expressions obtained show that a TI medium can be fully described from *P*- and *S*-wave traveltimes around only one axis of symmetry which is a point that, in my opinion, has not received considerable attention.

I start by rederiving the basic equations of the forward mapping from the expression of *P*- and *SV*-wave phase velocities in TI media. The calculations are done near both the horizontal and the vertical axes. Then, using these expressions, I solve the inverse mapping analytically. The final section illustrates the use and range of validity of the technique when estimating the elastic constants of a homogeneous medium from impulse responses sampled near either the vertical axis or the horizontal axis, to simulate VSP and crosswell configurations, respectively.

#### FORWARD MAPPING

By forward modeling, we create data from a given set of model parameters. If the data form a second, different set of model parameters that we also want to estimate, I prefer to call the process forward mapping to emphasize that the transformation is done between two sets of parameters rather than between parameters and measured data. In this section I explain how to do the mapping from the elastic constants of a TI media to different sets of phase velocities.

#### From elastic constants to phase velocities

The phase velocity expression for *P*- and *SV*-waves in TI media is (Auld, 1990)

$$2W_{P,SV}(\theta) = (W_{33} + W_{44}) \cos^2 \theta + (W_{11} + W_{44}) \sin^2 \theta \\ \pm \sqrt{[(W_{33} - W_{44}) \cos^2 \theta - (W_{11} - W_{44}) \sin^2 \theta]^2 + 4(W_{13} + W_{44})^2 \sin^2 \theta \cos^2 \theta}, \quad (1a)$$

two requirements when estimating elastic constants from either VSP or crosswell geometries.

The calculations presented here are valid for homogeneous media. When the model is heterogeneous, it can be described as a superposition of homogeneous regions, and the elliptical parameters needed at each region can be estimated tomographically, as explained by Michelena (1992) and Michelena et al. (1993). Once the elliptical parameters at each cell are estimated, the procedure developed here for homogeneous media can be applied at each cell to obtain 2-D maps of elastic constants.

where  $W(\theta)$  is the phase velocity squared and  $\theta$  is the phase angle from the vertical.  $W_{ij}$  is the  $(ij)$ th elastic modulus divided by density, with units of velocity squared. I refer to the quantity  $W_{ij}$  as an "elastic constant" in the remainder of this paper. The plus sign (+) in front of the square root corresponds to *P*-waves and the minus sign (-) to *SV*-waves. For *SH*-waves, the expression for the phase velocity is (Auld, 1990)

$$W_{SH}(\theta) = W_{44} \cos^2 \theta + W_{66} \sin^2 \theta. \quad (1b)$$

Near the vertical and horizontal axes equation (1a) is elliptical. The velocities that describe the corresponding ellipses are called elliptical velocities. In the next two subsections, I rederive equations contained in previous papers (Levin, 1979, 1980; Muir, 1990b) that are needed to calculate these elliptical velocities from the elastic constants. The expressions that result are used later in the paper to solve the inverse mapping when the data have narrow aperture.

#### From elastic constants to velocities near the vertical axis

Expanding equation (1a) around  $\theta = 0$  and neglecting terms in  $\sin^4 \theta$ , results in:

$$2W_{P,SV}(\theta) = (W_{33} + W_{44}) \cos^2 \theta + (W_{11} + W_{44}) \sin^2 \theta \\ \pm \left( (W_{33} - W_{44}) \cos^2 \theta - (W_{11} - W_{44}) \sin^2 \theta \right. \\ \left. + \frac{2(W_{13} + W_{44})^2}{W_{33} - W_{44}} \sin^2 \theta \right). \quad (2)$$

Choosing the positive root yields the *P*-wave phase velocity near the vertical axis, as follows:

$$W_P(\theta) = W_{P,z} c^2 + W_{P,xNMO} s^2, \quad (3)$$

where  $c = \cos \theta$ ,  $s = \sin \theta$ ,

$$W_{P,z} = W_{33}, \quad (4)$$

and

$$W_{P,xNMO} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}}. \quad (5)$$

$W_{P,z}$  is the vertical *P*-wave phase velocity squared and  $W_{P,xNMO}$  is the horizontal normal moveout (NMO) phase velocity squared.

Choosing the negative root in equation (2) yields *SV*-wave phase velocities near the vertical axis, as follows:

$$W_{SV}(\theta) = W_{SV,z} c^2 + W_{SV,xNMO} s^2, \quad (6)$$

where

$$W_{SV,z} = W_{44}, \quad (7)$$

and

$$W_{SV,xNMO} = W_{11} - \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}}. \quad (8)$$

The previous expressions for the NMO velocities agree with the results of Thomsen (1986) and Vernik and Nur (1992).

The expression for *SH*-wave phase velocities near the vertical axis is

$$W_{SH}(\theta) = W_{SH,z} c^2 + W_{SH,xNMO} s^2, \quad (9)$$

where

$$W_{SH,z} = W_{44}, \quad (10)$$

and

$$W_{SH,xNMO} = W_{SH,z} = W_{66}. \quad (11)$$

#### From elastic constants to velocities near the horizontal axis

When the phase velocities are parameterized in terms of the elastic constants, *P*- and *SV*-wave phase velocities near the horizontal axis can be obtained easily by interchanging  $W_{11}$  and  $W_{33}$  and  $c^2$  and  $s^2$  wherever they occur in equations (3) through equation (8). Thus, for *P*-waves the result is

$$W_P(\theta) = W_{P,z} s^2 + W_{P,xNMO} c^2, \quad (12)$$

where

$$W_{P,x} = W_{11}, \quad (13)$$

and

$$W_{P,zNMO} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}}. \quad (14)$$

For *SV*-waves, the expression for the phase velocity near the horizontal axis is

$$W_{SV}(\theta) = W_{SV,z} s^2 + W_{SV,xNMO} c^2, \quad (15)$$

where

$$W_{SV,x} = W_{44}, \quad (16)$$

and

$$W_{SV,zNMO} = W_{33} - \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}}. \quad (17)$$

Near the horizontal axis the *SH*-wave phase velocities are

$$W_{SH}(\theta) = W_{SH,z} s^2 + W_{SH,xNMO} c^2, \quad (18)$$

where

$$W_{SH,x} = W_{66}, \quad (19)$$

and

$$W_{SH,zNMO} = W_{SH,z} = W_{44}. \quad (20)$$

In the rest of the paper I refer to  $W_{P,x}$ ,  $W_{P,z}$ ,  $W_{P,xNMO}$ ,  $W_{P,zNMO}$ ,  $W_{SV,x}$ ,  $W_{SV,z}$ ,  $W_{SV,xNMO}$ ,  $W_{SV,zNMO}$ ,  $W_{SH,x}$ ,  $W_{SH,z}$ ,  $W_{SH,xNMO}$ , and  $W_{SH,zNMO}$  as  $W_*$ , direct or NMO phase velocity squared for *P*-, *SV*-, and *SH*-waves.

#### OBTAINING DIRECT AND NMO VELOCITIES

The expressions derived in the preceding section can be used to solve for the elastic constants as a function of the phase velocities. However, only in a few cases can phase velocities be estimated easily from traveltime measurements that give, instead, group velocities. The correspondence between phase and group velocities is trivial only when the model is isotropic or elliptically anisotropic, or when the ray travels along an axis of symmetry.

Equations (3), (6), (12), and (15) show that the phase velocities of *P*- and *SV*-waves are elliptical near the axes of symmetry. The *SH*-wave velocities are also elliptical [equation (1b)]. It has been shown that when the phase

velocity has an elliptical shape, the corresponding impulse response is also elliptical (Levin, 1978; Byun, 1982). Therefore, the group slowness expression that corresponds to these equations has the general form

$$S^2(\phi) = S_z^2 \cos^2 \phi + S_x^2 \sin^2 \phi, \quad (21)$$

where  $\phi$  is the ray angle measured from the vertical and  $S_*$  (the ray slowness) is

$$S_*^2 = \frac{1}{W_*}, \quad (22)$$

which means that phase and group-direct or NMO velocities are equal.

To estimate  $S_*$ , I use the expression for the traveltime of a ray that travels a distance  $\ell = \sqrt{\Delta x^2 + \Delta z^2}$  between two points:

$$t^2 = \Delta x^2 S_x^2 + \Delta z^2 S_z^2. \quad (23)$$

This equation, which has the same form as the isotropic moveout equation, is obtained after multiplying equation (21) by  $\ell^2$ . Velocities estimated from the moveout near one axis using this traveltime equation are called NMO velocities, and velocities estimated from arrival times along the same axis are called direct velocities. Hence the different names chosen for the phase velocities  $W_*$ .

#### INVERSE MAPPING

The equations derived in the section "Forward Mapping," which relate direct and NMO velocities to elastic constants, suggest several approaches to solve the inverse mapping that depend on the recording geometry, the recording aperture, and the wave types available. From  $SH$ -wave velocities near either axis, it is always possible to estimate  $W_{44}$  and  $W_{66}$  because the NMO velocities are equal to the corresponding direct velocities. However, from  $P$ - and  $SV$ -wave velocities near either axis, the estimation of the corresponding elastic constants is not straightforward. This section explains what to do in such a case.

#### Using $P$ - and $SV$ -wave full aperture velocities

To estimate  $W_{11}$ ,  $W_{33}$ ,  $W_{44}$ , and  $W_{13}$  directly from equation (1a), we need at least four phase velocities at four different angles between 0 and 90 degrees. This is a well-studied problem (Dellinger and Vernik, 1992).  $W_{ij}$  is the solution of a system of nonlinear equations where the independent term is formed by these phase velocities. Along the axes, the system of equations is almost diagonal and the estimation of three elastic constants ( $W_{33}$ ,  $W_{11}$ , and  $W_{44}$ ) is straightforward:

$$W_{33} = W_{P,z}, \quad (24a)$$

$$W_{11} = W_{P,x}, \quad (24b)$$

$$W_{44} = W_{SV,z} = W_{SV,x}. \quad (24c)$$

The elastic constants are estimated directly from velocities along the axes.  $W_{13}$  can be estimated from the previous elastic constants and one phase velocity at an oblique angle, typically 45 degrees.

This approach, although simple in theory, is not applicable in many practical situations because wide aperture data are required (the angles of the observations must include 0, 90 degrees, and one intermediate angle) to simplify the system of equations. This requirement does not exist for most single-geometry data sets (either surface, or crosswell or VSP) where no rays travel along (at least) one of the axes. Therefore, velocities along both axes cannot be estimated without having to assume a velocity symmetry (e.g., isotropic, elliptical, TI).

#### Using $P$ - and $SV$ -wave narrow aperture velocities

**Elastic constants from velocities near the vertical.**—When the phase angles are close to zero, it is possible to estimate elastic constants from the corresponding phase velocities by using equations (4), (5), (7), and (8), a system of four equations and four unknowns. The independent term is formed by  $W_{P,z}$ ,  $W_{P,xNMO}$ ,  $W_{SV,z}$ , and  $W_{SV,xNMO}$ . The solution of this system of equations is

$$W_{33} = W_{P,z}, \quad (25a)$$

$$W_{44} = W_{SV,z}, \quad (25b)$$

$$W_{13} = \sqrt{(W_{P,xNMO} - W_{SV,z})(W_{P,z} - W_{SV,z})} - W_{SV,z}, \quad (25c)$$

$$W_{11} = W_{SV,xNMO} + W_{P,xNMO} - W_{SV,z}. \quad (25d)$$

In equation (25d),  $W_{11}$  is a linear combination of elliptical parameters independent of the horizontal  $P$ -wave velocity, unlike  $W_{11}$  estimated from equation (24b). Roughly speaking, equation (25d) indicates that summing  $P$ - and  $SV$ -wave NMO velocities (squared) is the same as summing the elastic constants that control the horizontal wave propagation. The examples in the final section of this paper show the range of angles near the vertical for which these equations are valid.

Since this approximation simultaneously uses the elliptical parameters of two ellipses fitted near the vertical, I call it *vertical double-elliptic approximation*, analogous to Muir's double-elliptic approximation that uses horizontal and vertical ellipses to approximate the slowness surface and the impulse response for all angles (Muir, 1990a; Dellinger et al., 1993). It is important to point out the differences between these two approximations. On the one hand, the *vertical double-elliptic approximation* is used to estimate elastic constants from velocities near the vertical axis. Slowness surfaces and impulse responses can be calculated from these elastic constants using equation (1a) and the exact relationships between phase and group velocities. On the other hand, Muir's double-elliptic approximation is used to directly estimate slowness surfaces and impulse responses from data near both axes without having to know the elastic constants.

When the elliptical fitting is done near the horizontal axis, the result is the *horizontal double-elliptic approximation*, as follows.

**Elastic constants from velocities near the horizontal.**—When the phase angle is close to 90 degrees, the expressions for the elastic constants as a function of  $P$ - and  $SV$ -wave direct and NMO velocities are obtained by solving the

system of equations (13), (14), (16), and (17), with the following result:

$$W_{11} = W_{P,x}, \quad (26a)$$

$$W_{44} = W_{SV,x}, \quad (26b)$$

$$W_{13} = \sqrt{(W_{P,zNMO} - W_{SV,x})(W_{P,x} - W_{SV,x})} - W_{SV,x}, \quad (26c)$$

$$W_{33} = W_{SV,zNMO} + W_{P,zNMO} - W_{SV,x}. \quad (26d)$$

This set of equations forms the *horizontal double-elliptic approximation*. It uses direct and NMO *P*- and *SV*-wave velocities around the horizontal to approximate the elastic constants. Notice that the estimation of  $W_{33}$  is independent of the *P*-wave velocity along the vertical axis. Equations (26) can be obtained easily from equation (25) by interchanging  $W_{11}$  and  $W_{33}$ , and  $x$  and  $z$ , which is the main advantage of parameterizing the velocities directly in terms of the elastic constants.

The estimation of  $W_{33}$  from near-horizontal velocities [equation (26d)] and  $W_{11}$  from near-vertical velocities [equation (25d)] is in both cases the sum of NMO velocities minus  $W_{44}$ . Michelena et al. (1993) and Michelena (1993) show that when estimating velocities tomographically, NMO velocities correspond to the smallest singular values of the problem. The largest singular values correspond to velocities estimated from rays that travel along the axes. Therefore, as expected, estimating  $W_{33}$  from crosswell traveltimes alone is a harder problem than estimating  $W_{11}$  from the same data. The opposite is true when estimating  $W_{33}$  and  $W_{11}$  from VSP measurements.

#### Using only *P*-wave velocities near the axes

When the medium is TI and no information is available about *SV*-wave velocities, it is still possible to obtain four elastic constants from *P*-wave velocities alone near both axes. This is done by solving the system of equations (4), (5), (13), and (14), which yields:

$$W_{11} = W_{P,x}, \quad (27a)$$

$$W_{33} = W_{P,z}, \quad (27b)$$

$$W_{44} = \frac{W_{P,xNMO}W_{P,z} - W_{P,zNMO}W_{P,x}}{W_{P,xNMO} + W_{P,z} - W_{P,zNMO} - W_{P,x}}, \quad (27c)$$

$$W_{13} = \sqrt{(W_{P,zNMO} - W_{44})(W_{P,x} - W_{44})} - W_{44}. \quad (27d)$$

This set of equations forms the *P*-wave double-elliptic approximation of the elastic constants in a TI medium.

In this approximation, as well as in the previous ones, the assumption of transverse isotropy is crucial. When the medium is isotropic ( $W_{P,x} = W_{P,z} = W_{P,xNMO} = W_{P,zNMO}$ ) there is no way to calculate  $W_{44}$  (the shear moduli) from *P*-wave velocities alone because equation (27c) is indeterminate. When the medium is weakly anisotropic, the estimation of  $W_{44}$  using this approximation may still be unreliable because both the numerator and the denominator in equation (27c) are close to zero. We will see later in the examples that even when the medium is moderately anisotropic this approximation breaks down quickly for phase

angles not close to the axes, unlike the previous vertical and horizontal double-elliptic approximations [equations (25) and (26), respectively] that have a wider range of validity. When the medium is weakly anisotropic, it may still be possible to obtain information about a combination of  $W_{44}$  and  $W_{13}$  by parameterizing the velocities as proposed by Thomsen (1986).

If only *SV*-wave velocities near the axes are available, it is not possible to obtain the corresponding elastic constants because the system of equations (7), (8), (16), and (17) is underdetermined.

#### USING TRAVELTIMES TO ESTIMATE ELASTIC CONSTANTS

This section summarizes the steps needed to obtain the elastic constants of a TI medium from constrained aperture traveltimes measurements.

##### Homogeneous medium

The procedure to estimate the elastic constants of a homogeneous TI medium from traveltimes measurements near one axis of symmetry is the following:

- 1) Fit the traveltimes with elliptically anisotropic models, one model for each wave type. This gives direct and NMO velocities.
- 2) From the estimated direct and NMO velocities, find the elastic constants using the equations that correspond to the given recording geometry [either equations (25) or (26)]. In the case of *SH* waves, the estimated velocities squared are the same as the corresponding elastic constants.

##### Heterogeneous medium

To estimate spatial variations in the elastic constants, the first step is to describe the model as a superposition of homogeneous blocks that incorporate our previous knowledge about the structure. Then, the elliptical slownesses of the different wave types are estimated tomographically (Michelena et al., 1993) by solving a system of nonlinear equations of the form

$$t_i = \sum_{j=1}^N \sqrt{\Delta x_{ij}^2 S_{xj}^2 + \Delta z_{ij}^2 S_{zj}^2} \quad i = 1, \dots, M,$$

where  $t_i$  are the measured traveltimes, and  $S_{xj}$  and  $S_{zj}$  are the horizontal and vertical slownesses, respectively, in the  $j$ th block.  $\Delta x_{ij}$  and  $\Delta z_{ij}$  are the horizontal and vertical distances traveled by the  $i$ th ray in the  $j$ th block.  $M$  is the total number of traveltimes, and  $N$  is the total number of blocks. Finally, local transformations from these elliptical velocities to elastic constants are performed at each homogeneous block by using the procedure for homogeneous media explained in the preceding section.

Estimation of elastic constants from narrow-aperture traveltimes and tomographic estimation of direct and NMO velocities have opposite requirements in terms of data aperture. On the one hand, the estimation of elastic constants requires traveltimes from rays that travel as close as possible to one axis [this is how equations (25) and (26) were derived].

On the other hand, the tomographic estimation of direct and NMO velocities requires wide ray angles to improve the conditioning of the problem, the accuracy of the NMO velocities, and the spatial resolution of the result. Fortunately, as the examples will show, there is a common range of ray angles in which accurate results can be obtained with both procedures. However, the accuracy will depend in general on the complexity of the heterogeneity. Problems can be anticipated if the velocity varies rapidly.

#### Heterogeneous medium with nonvertical axes of symmetry

So far, I have assumed that the axis of symmetry is vertical. If the axis of symmetry is nonvertical we need to find its inclination first, for example, by fitting *SH*-wave traveltimes with heterogeneous elliptically anisotropic models, as explained in Michelena (1992). Once the inclination of the axes of symmetry of the different blocks has been estimated, *P*- and *SV*-wave direct and NMO velocities at each block are estimated using only rays that travel near the axes of symmetry. Finally, the direct and NMO velocities are transformed locally into elastic constants, as explained in the section "Homogeneous Medium." This process assumes the axes of symmetry of the different blocks of the model are in the same plane of the survey, as explained also by Michelena (1992).

The elastic constants estimated in this way are referred to different coordinate frames, one for each different axis of symmetry. For purposes of interpretation, having the elastic constants relative to different frames is not a problem as long as we also use the inclination of the axes of symmetry. However, for further computations (finite-difference modeling, for example) it might be necessary to transform the elastic constants to a common frame. This can be done by using Bond's transformation matrices (Auld, 1990).

#### EXAMPLES

This section contains examples that illustrate how four elastic constants of a homogeneous TI medium can be estimated when the data are (1) *P*- and *SV*-wave traveltimes from VSP geometries with different apertures, (2) *P*- and *SV*-wave traveltimes from crosswell geometries with different apertures, and (3) *P*-wave traveltimes from crosswell and VSP geometries with different apertures.

The impulse response is given, and it is the same for all examples. The elastic constants that describe the medium are:  $W_{11} = 2256^2$ ,  $W_{33} = 1919^2$ ,  $W_{13} = 1699^2$ , and  $W_{44} = 658^2$ , all with units of  $(\text{m/s})^2$  (Byun and Corrigan, 1990). I calculate the ellipse that fits the impulse response near the chosen axis by using a ray along the axis and a ray close to the axis. Once I have found the four needed elliptical parameters I use equations (25), (26), or (27) to convert those elliptical parameters into elastic constants.

#### Estimating $W_{ij}$ from VSP *P*- and *SV*-wave traveltimes

Figure 1 shows how the vertical double-elliptic approximation works for different angles around the vertical. The parameters of the ellipses that approximate the impulse response have been calculated at the angles shown by the straight lines. Column (a) compares the given impulse re-

sponses for *P*- and *SV*-waves (continuous lines) with the elliptical approximations around the vertical (dashed lines). With the four elliptical parameters obtained for each aperture, I calculate the elastic constants of the medium by using equation (25). From the estimated elastic constants, I calculate the corresponding impulse-responses for both *P*- and *SV*-waves. The result is shown in column (b) (dashed lines) simultaneously with the given impulse responses. In most cases (except at 40-degree aperture), the agreement is excellent. At 40-degree aperture the horizontal *P*-wave group velocity has been overestimated and the shear-wave tripllication is larger than expected. Column (c) shows the absolute value of the percentage error made in the estimation of the elastic constants. For small angles ( $\leq 10$  degrees), the error is negligible. For angles between 10 and 30 degrees, the error is smaller in  $W_{11}$  (A) than in  $W_{13}$  (F). At large angles the error in  $W_{11}$  (A) is the largest, almost 30 percent. Notice that up to 30 degrees, even though the error in the estimation of the elastic constants is not zero but a few percent, the differences between the given and the estimated impulse responses are hard to see.

In Figure 1, the elliptical approximations to the impulse responses are calculated using two ray angles at a time, one zero and one nonzero. In Figure 2, I show what happens when all these angles (or different VSP offsets) are used simultaneously to calculate the elliptical approximations and the elastic constants. The horizontal *P*-wave group velocity is now slightly overestimated and the shear velocities are retrieved well. The errors in the estimated elastic constants are 6 percent in  $W_{11}$  (A) and 4 percent in  $W_{13}$  (F). The errors made when using only large angles have now been compensated by also using small angles.

#### Estimating $W_{ij}$ from crosswell *P*- and *SV*-wave traveltimes

Figure 3 shows how the horizontal double-elliptic approximation works for different ray angles measured from the horizontal. When comparing this case with the VSP case (Figure 1), we see that the approximation works better for ray angles around the horizontal than around the vertical, even when the angles are large. The estimated impulse responses agree well with the given ones, and the errors in the elastic constants are negligible for angles less than 30 degrees.

The reason why the horizontal double-elliptic approximation works better than the vertical double-elliptic approximation is that the elliptical approximation for the *P*-wave impulse response is more adequate at larger angles around the horizontal than around the vertical, where the group velocity is lower. In this example, the elliptic approximations around the vertical are almost circular.

Figure 4 shows that when small and large ray angles are used simultaneously to calculate the elliptical approximations around the horizontal, the result is roughly an average of the results shown in Figure 3. The agreement between the given and the estimated impulse responses is excellent, and the error in the estimation of the elastic constants is  $\approx 2$  percent for  $W_{13}$  (F) and  $\approx 1$  percent for  $W_{33}$  (C).

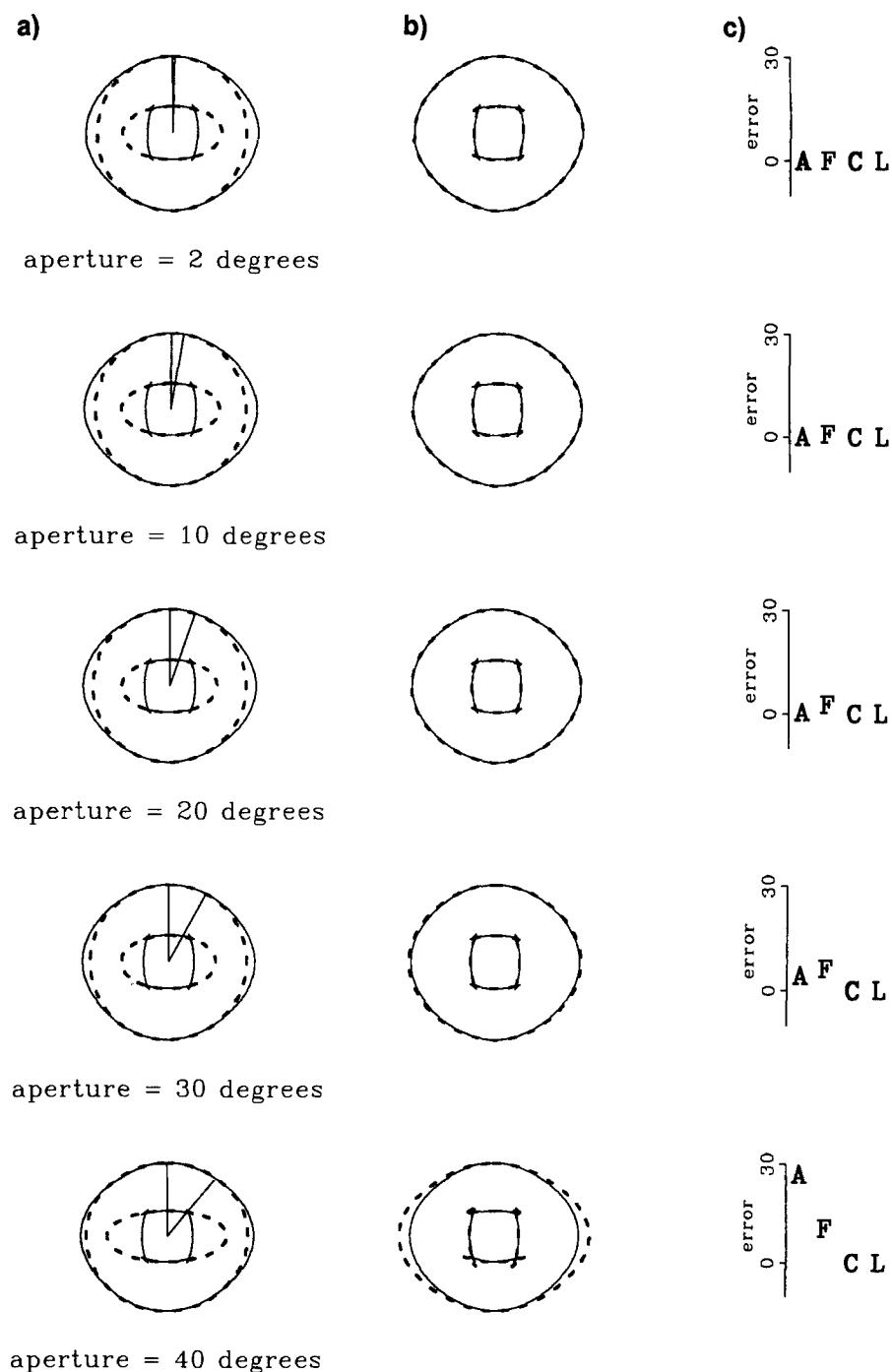


FIG. 1. (a) Impulse response for  $P$ - and  $SV$ -waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). (b) Given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). (c) Absolute value of the percentage error made in the estimation of the elastic constants when using the vertical, double-elliptic approximation. The elastic constants are  $A = W_{11}$ ,  $F = W_{13}$ ,  $C = W_{33}$ , and  $L = W_{44}$ . The density is assumed to be unity.

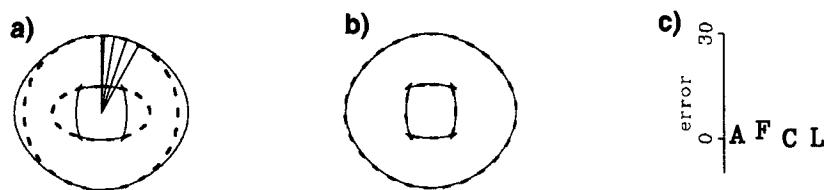


FIG. 2. (a) Impulse response for  $P$ - and  $SV$ -waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). All ray angles shown are used simultaneously to calculate the elliptical approximations. (b) Given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). (c) Absolute value of the percentage error made in the estimation of the elastic constants when using the vertical, double-elliptic approximation. The elastic constants are  $A = W_{11}$ ,  $F = W_{13}$ ,  $C = W_{33}$ , and  $L = W_{44}$ .

### Estimating $W_{ij}$ from crosswell and VSP P-wave traveltimes

For TI media, the elastic constants that control the  $P$ - and  $SV$ -wave propagation can be estimated from  $P$ -wave measurements near the axes as equation (27) shows. Figure 5 shows an example. Column (a) shows the given  $P$ -wave impulse response (continuous line) and the approximating ellipses (dashed lines) around the horizontal and around the vertical for two different apertures. Column (b) shows the

given and approximate impulse responses for  $P$ - and  $SV$ -waves, the approximate impulse responses being calculated from the estimated elastic constants. Column (c) shows the absolute value of the error made in the estimation of the elastic constants. Unlike the previous approximations, the  $P$ -wave double-elliptic approximation is valid only at very small angles ( $\approx 2$  degrees). At greater angles it yields highly inaccurate results, particularly in the estimation of  $W_{44}$  (L), which controls the shear-wave propagation along the axes.

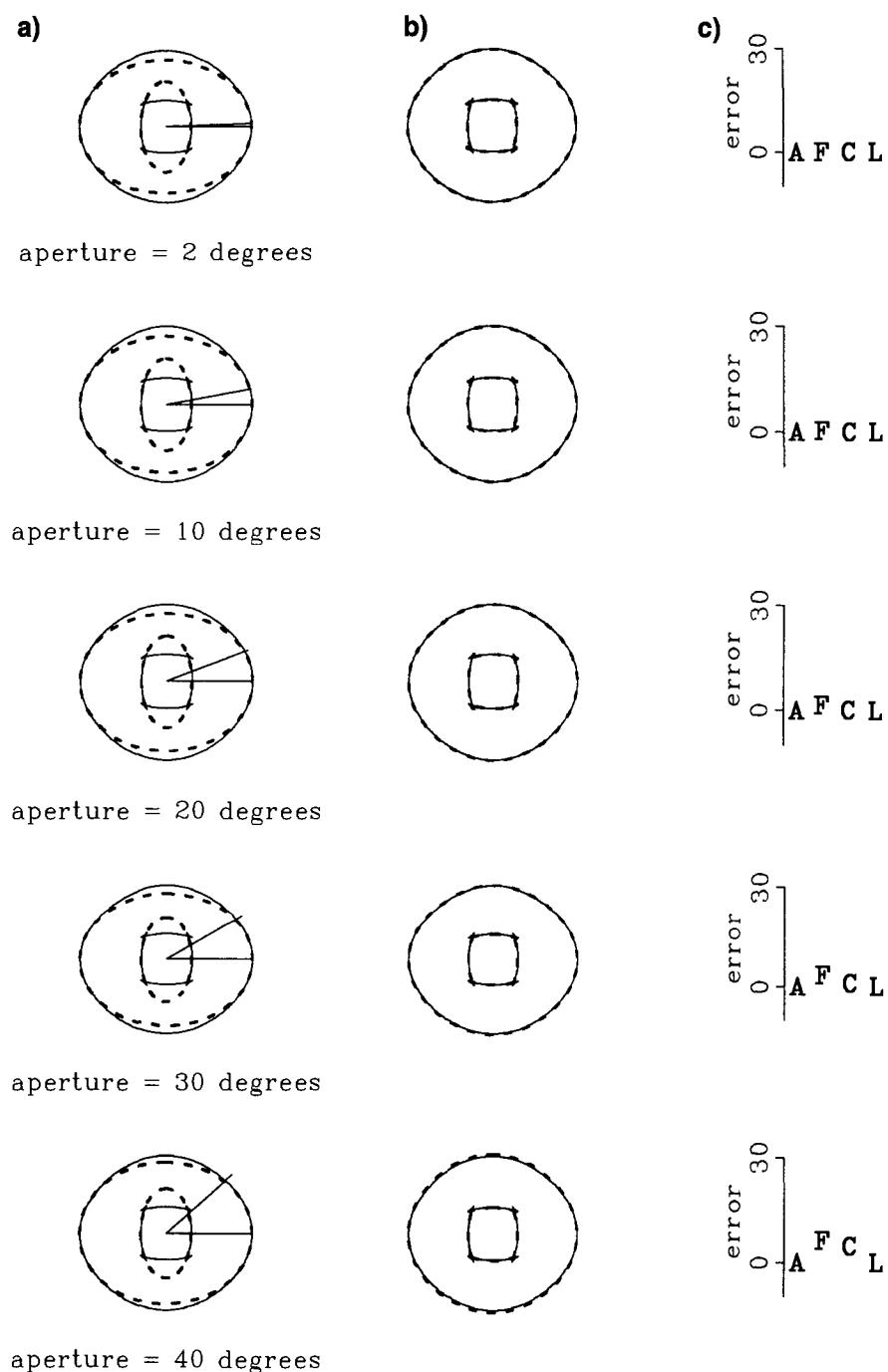


FIG. 3. (a) Impulse response for  $P$ - and  $SV$ -waves (continuous lines) compared with their elliptical approximations around the horizontal (dashed lines). (b) Given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). (c) Absolute value of the percentage error made in the estimation of the elastic constants when using the horizontal, double-elliptic approximation. The elastic constants are  $A = W_{11}$ ,  $F = W_{13}$ ,  $C = W_{33}$ , and  $L = W_{44}$ .

Note that for an aperture of 10 degrees, the shear-wave impulse response has been considerably underestimated.

This approximation may be hard to use in practical situations because it works only for very small angles near the axes.

### CONCLUSIONS

I have shown how to estimate the elastic constants of homogeneous TI media from *P*-, *SV*-, and *SH*-wave traveltimes near a single axis of symmetry (either from VSP or crosswell geometries). For heterogeneous TI media, the procedure can be easily generalized by using tomographic techniques (Michelena, 1992; Michelena et al., 1993).

The technique uses the parameters obtained by fitting traveltimes near one axis with elliptical models. For *SH*-wave traveltimes, the estimation of the corresponding TI elastic constants is trivial because *SH*-wave phase velocities are also elliptical in TI media. For *P*- and *SV*-wave traveltimes, four parameters are needed to estimate the phase velocities at all angles from measurements near one axis. These parameters are the direct and NMO phase velocities for *P*- and *SV*-waves. The transformation from NMO and

direct velocities to elastic constants is simple for both VSP and crosswell geometries.

The accuracy of the estimation of the elastic constants from the elliptical parameters depends on the accuracy of the estimation of the NMO velocities. For small ray angles the approximation is excellent when using either VSP or crosswell geometries, because the elliptical approximations are accurate in both cases. In practice, however, using only small ray angles may hinder an accurate estimation of NMO velocities. For intermediate ray angles, the estimation of vertical *P*-wave velocities from crosswell geometries is more accurate than the estimation of horizontal *P*-wave velocities from VSP. This is because the elliptical approximation for *P*-wave impulse response fits wider angles around the horizontal than around the vertical where the group velocity is usually smaller. For large ray angles the approximation does not work because elliptical fits are not as accurate.

When compressional and shear velocity logs are also available, they can be used either to add redundancy in the estimation of  $W_{11}$  and  $W_{44}$  or to check whether the assumption of transverse isotropy is valid, particularly when using

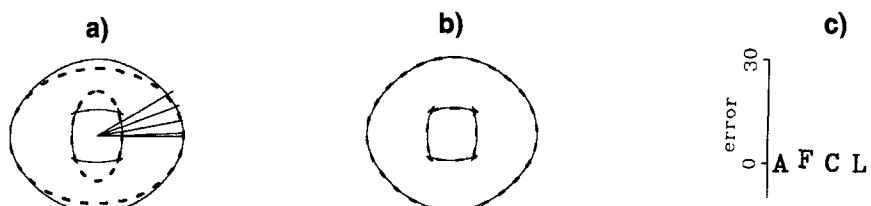


FIG. 4. (a) Impulse response for *P*- and *SV*-waves (continuous lines) compared with their elliptical approximations around the horizontal (dashed lines). All ray angles shown are used simultaneously to calculate the elliptical approximations. (b) Given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). (c) Absolute value of the percentage error made in the estimation of the elastic constants when using the horizontal, double-elliptic approximation. The elastic constants are  $A = W_{11}$ ,  $F = W_{13}$ ,  $C = W_{33}$ , and  $L = W_{44}$ .

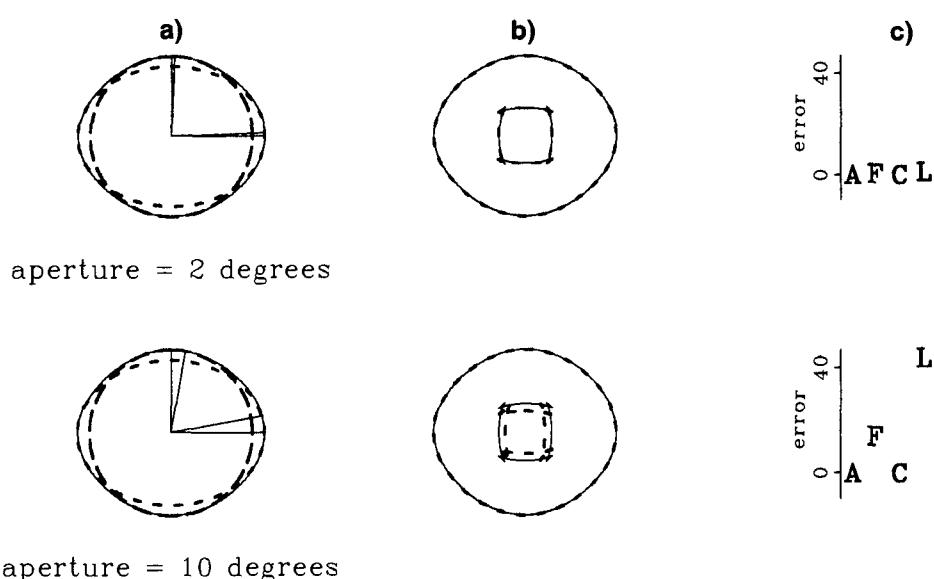


FIG. 5. *P*-wave impulse response used to estimate the elastic constants from measurements around the axes. (a) Given *P*-wave impulse response (continuous) and elliptical approximations around the axes (dashed). (b) Given impulse response (continuous) and estimated ones (dashed) for *P*- and *SV*-waves. (c) Absolute value of the percentage error made in the estimation of the elastic constants when using the *P*-wave, double-elliptic approximation. The elastic constants are  $A = W_{11}$ ,  $F = W_{13}$ ,  $C = W_{33}$ , and  $L = W_{44}$ .

crosswell measurements where vertical velocities are not well-sampled.

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